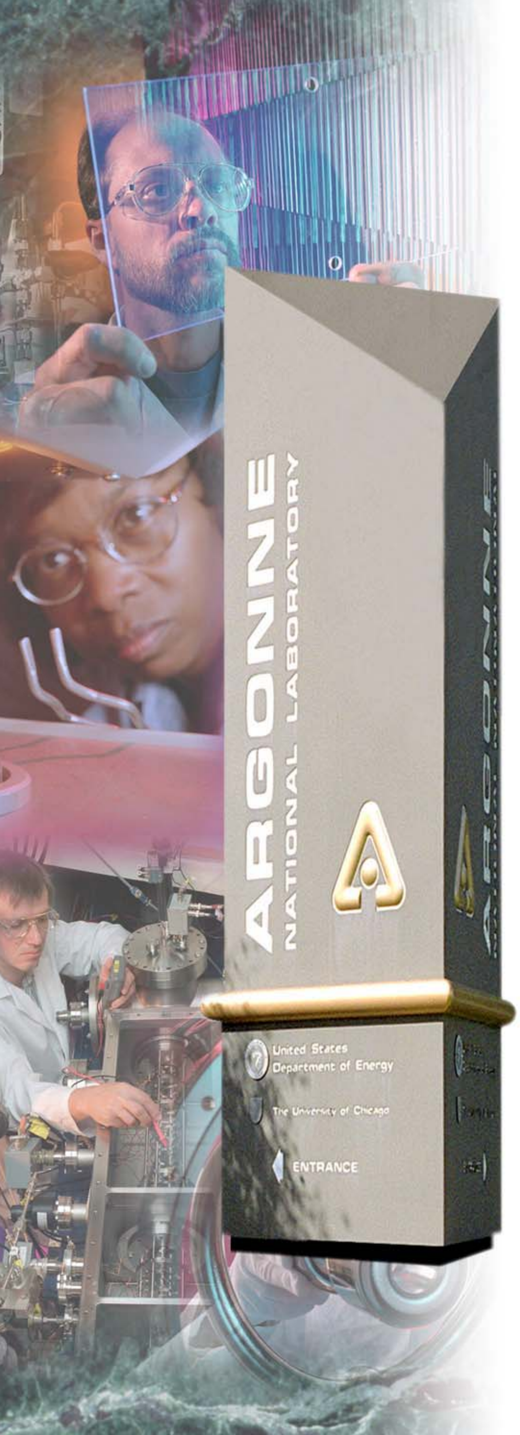

The Study of Off-momentum Particle Motion with Independent Component Analysis

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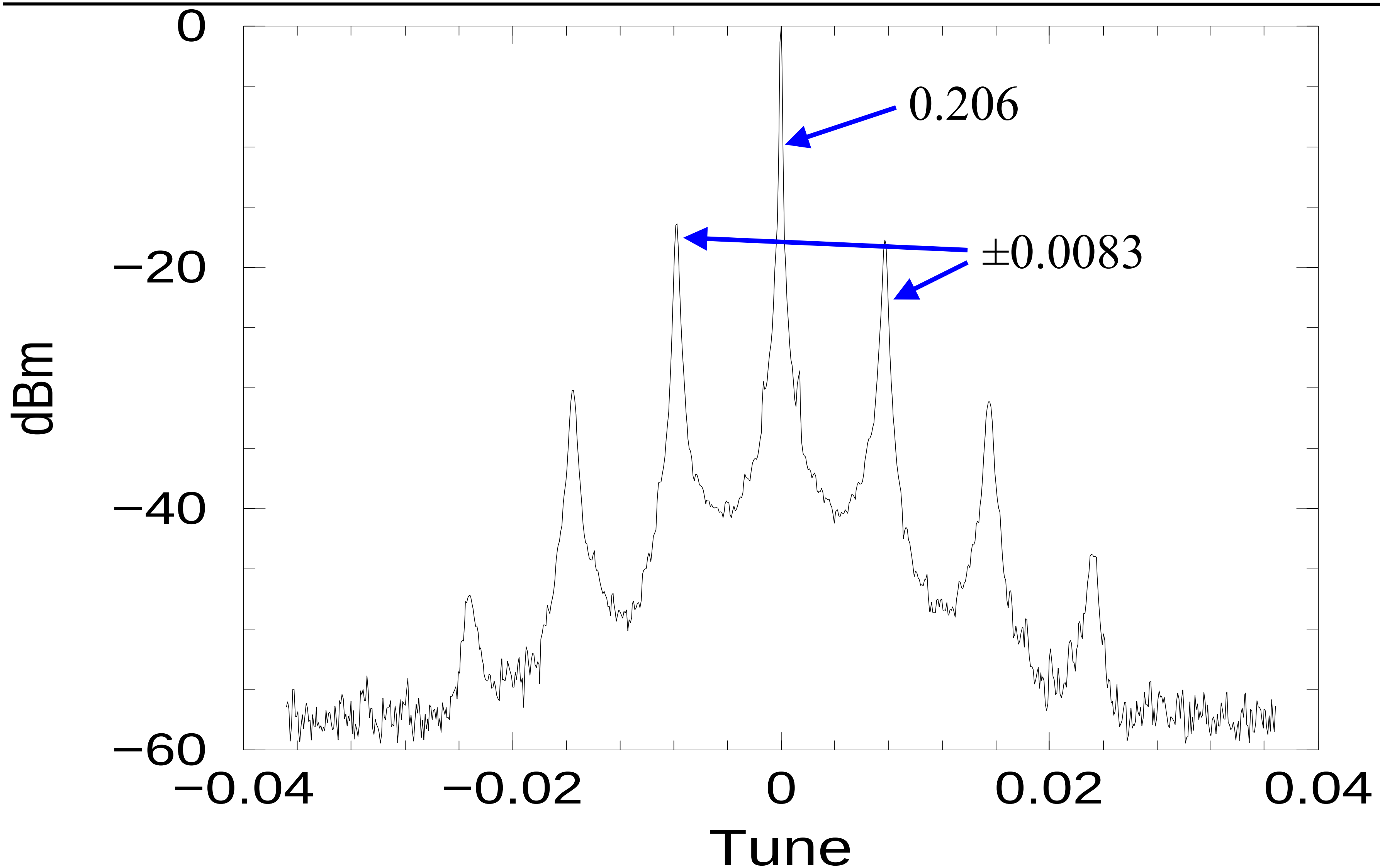
What is the mechanism of synchrotron sidebands?



The current theory of dispersion

The transverse motion Hamiltonian

$$H = -(1 + x/\rho)[P^2 - (p_x - eA_x)^2 - (p_y - eA_y)^2]^{1/2} - eA_s$$

Consider only quadrupoles and dipoles, the first order dispersion equation

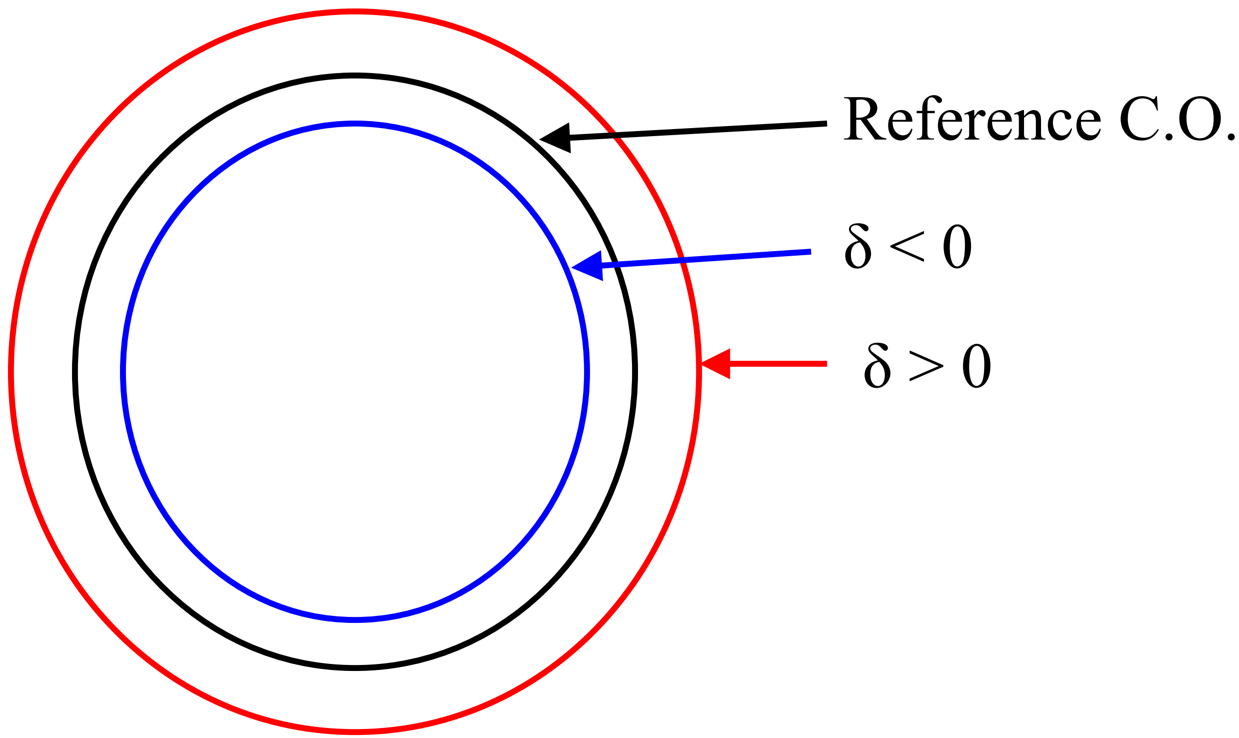
$$x'' + [(1 - 2\delta)/\rho^2 - K_1(1 - \delta)]x = \delta/\rho$$

Let $x = x_{c.o.} + x_\beta + D\delta$, the equation for the dispersion function

$$D'' + (1/\rho^2 - K_1)D = 1/\rho$$

The boundary condition $D(s) = D(s + C)$

The physical picture of dispersion



Analytical solution

$$x'' + K_\delta(s)x = \delta / \rho \qquad K_\delta = (1-2\delta)/\rho^2 - K_1(1-\delta)$$

In vector space

$$\frac{d}{ds}Z = \begin{pmatrix} 0 & 1 \\ -K_\delta & 0 \end{pmatrix} Z + F, \quad Z = \begin{pmatrix} x \\ x' \end{pmatrix}, \quad F = \begin{pmatrix} 0 \\ \delta / \rho \end{pmatrix}$$

The solution of the homogeneous equation

$$Z_h = M(s \mid 0) Z(0)$$

M(s|0) is the transformation matrix

The solution of the inhomogeneous equation

$$Z = M(s \mid 0)[Z(0) + \int_0^s M^{-1}(s_1) F(s_1) ds_1]$$

The dispersion functions

$$x = x_\beta + x_\delta$$

$$\begin{cases} x_\delta = \eta_x(s)\delta + I_D(s)\sin(\nu_x\theta + \chi(s))\delta \\ x'_\delta = \eta'_x(s)\delta - \sqrt{\dfrac{\gamma_\delta}{\beta_\delta}} I_D(s)\sin(\nu_x\theta + \chi(s) - \chi_0(s))\delta \end{cases}$$

$$\begin{cases} \eta_x(s) = \dfrac{\sqrt{\beta_\delta(s)}}{2\sin\pi\nu_x}\displaystyle\int_s^{s+C} ds_1 \dfrac{\sqrt{\beta_\delta(s_1)}}{\rho(s_1)}\cos\phi(s_1 \mid s) \\ \eta'_x(s) = \dfrac{-1}{2\sqrt{\beta_\delta(s)}\sin\pi\nu_x}\displaystyle\int_s^{s+C} ds_1 \dfrac{\sqrt{\beta_\delta(s_1)}}{\rho(s_1)}[\sin\phi(s_1 \mid s) \\ \qquad + \alpha_\delta(s_1)\cos\phi(s_1 \mid s)] \end{cases}$$

The $I_D(s)$ and $\chi(s)$ function

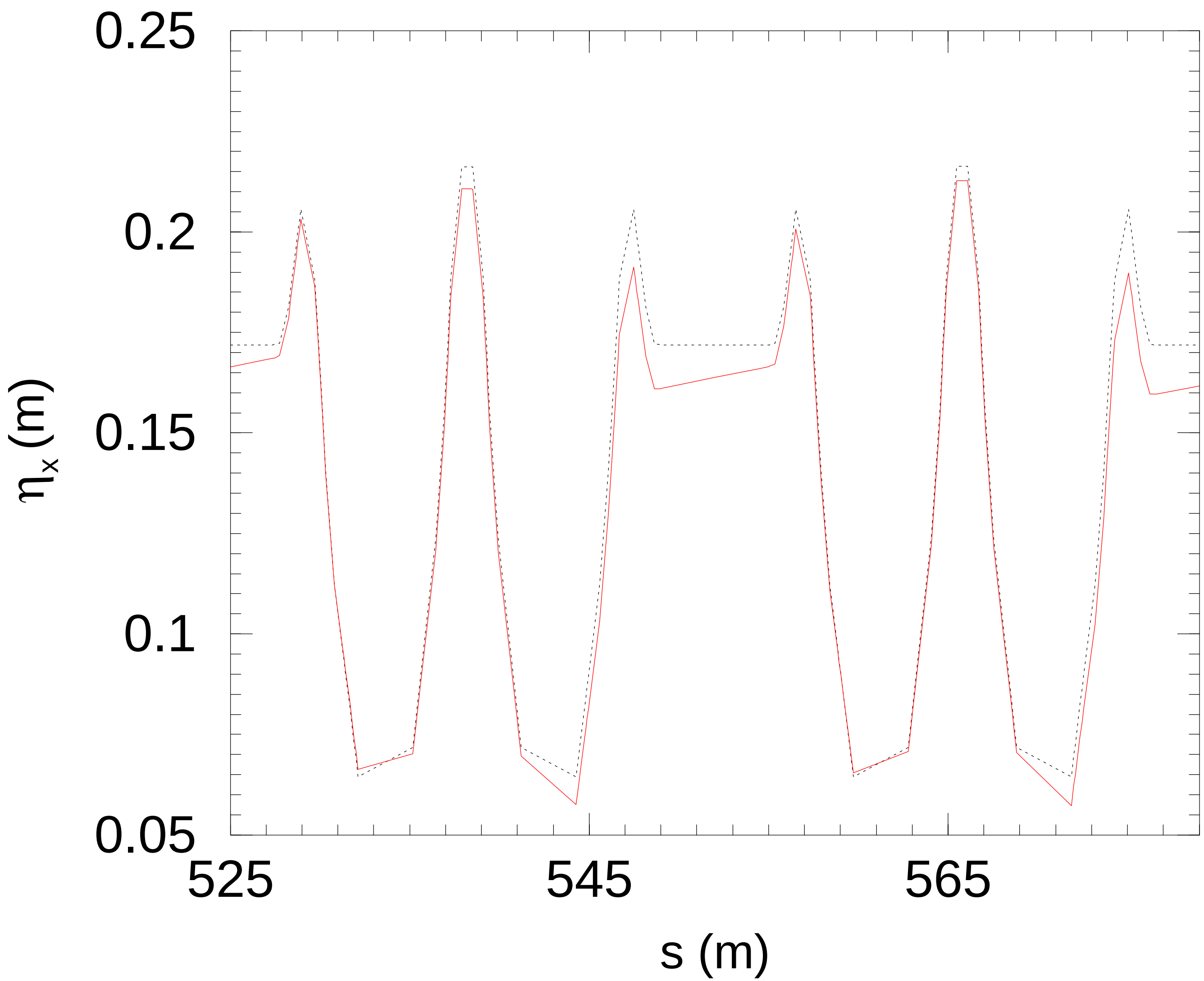
$$I_D(s) = \sqrt{A^2 + B^2}, \chi(s) = \tan^{-1} A/B$$

$$A = J_s(x) - \eta_x(s), B = J_c(s) - \alpha_\delta(s)\eta_x(s) - \beta_\delta(s)\eta'_x(s)$$

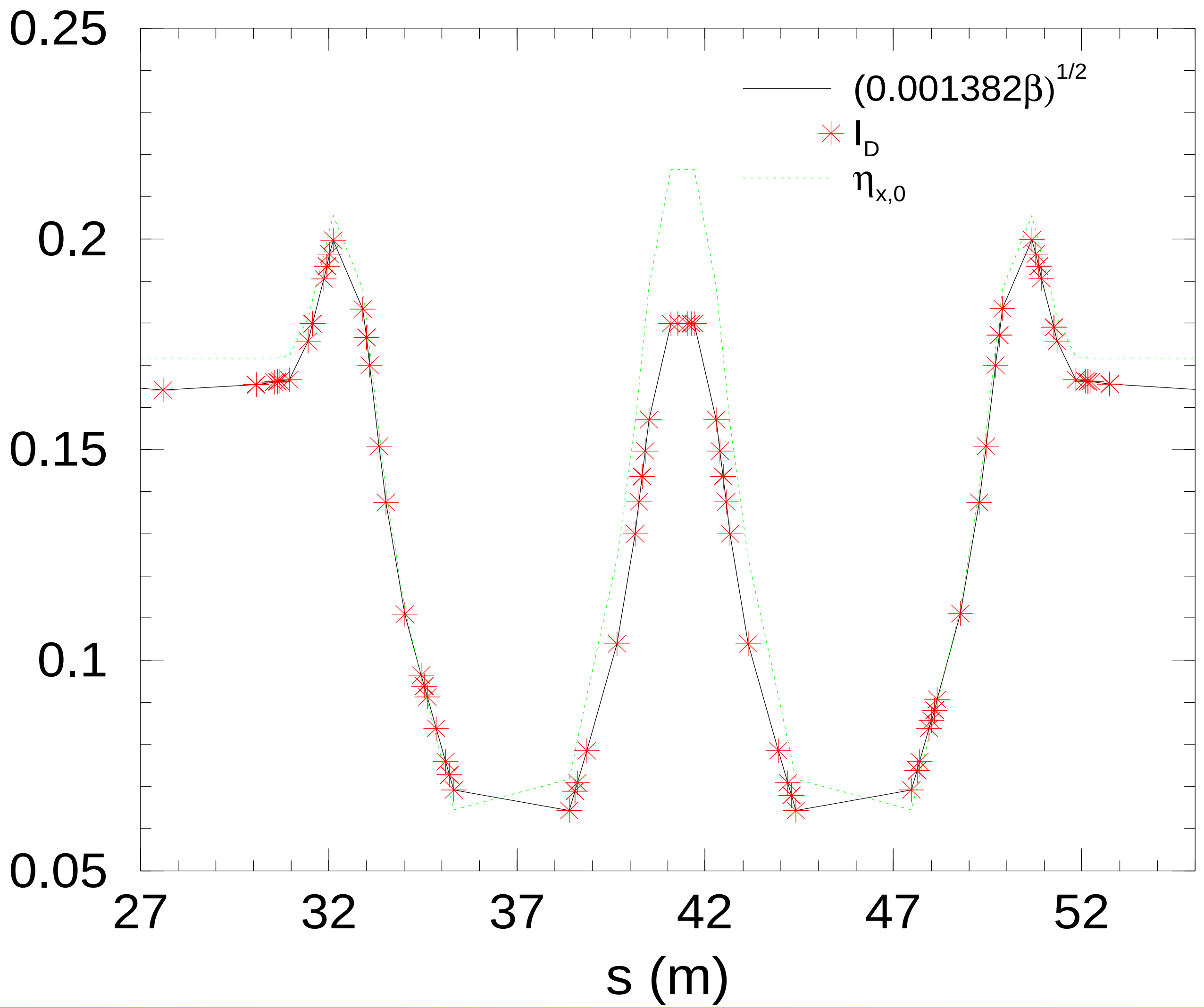
$$\begin{cases} J_s(s) = \sqrt{\beta_\delta(s)} \int_0^s ds_1 \dfrac{\sqrt{\beta_\delta(s_1)}}{\rho(s_1)} \sin(\psi(s) - \psi(s_1)) \\ J_c(s) = \sqrt{\beta_\delta(s)} \int_0^s ds_1 \dfrac{\sqrt{\beta_\delta(s_1)}}{\rho(s_1)} \cos(\psi(s) - \psi(s_1)) \end{cases}$$

$$I'_D(s) = -\frac{\alpha_\delta(s)}{\beta_\delta(s)} I_D(s), \chi'(s) = \frac{1}{\beta_\delta(s)}$$

The calculated dispersion function

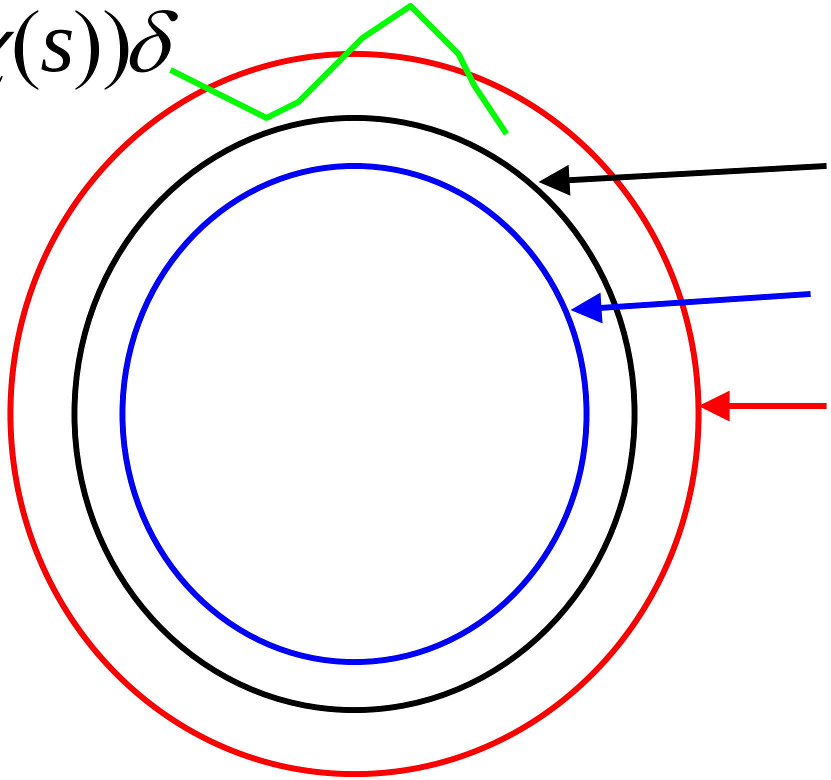


The calculated I_D function



The meaning of the I_D term

$$x_\delta = \eta_x(s)\delta +$$
$$I_D(s)\sin(\nu_x\theta + \chi(s))\delta$$



The higher order terms

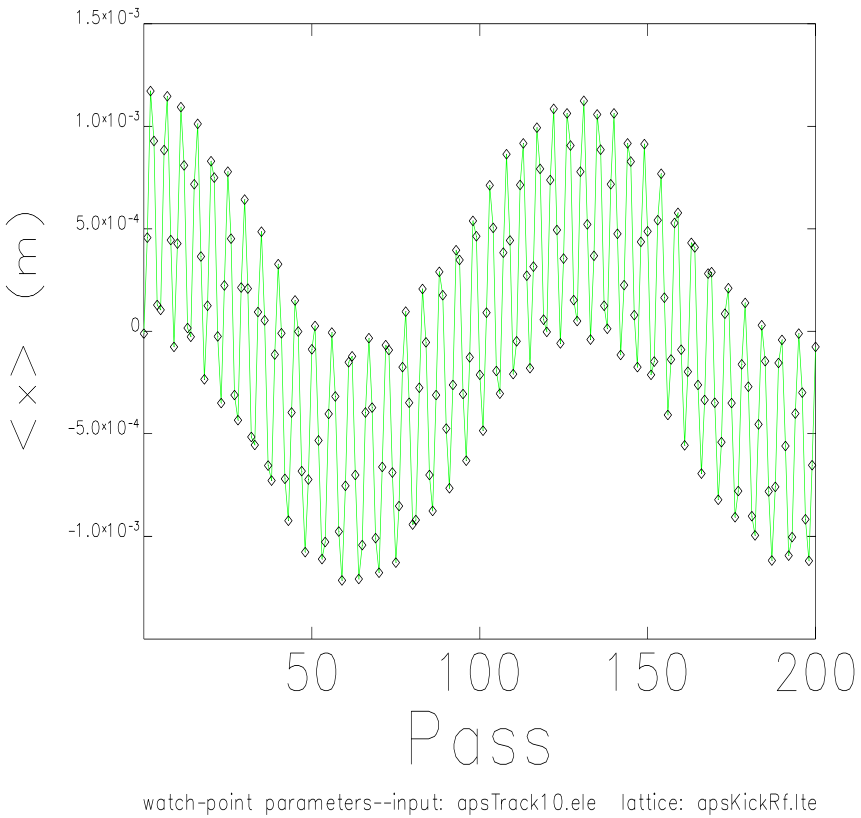
$$x'' + \left[\frac{1-\delta}{\rho^2(1+\delta)} - \frac{K_1}{1+\delta} \right] x = \frac{\delta}{\rho(1+\delta)}$$

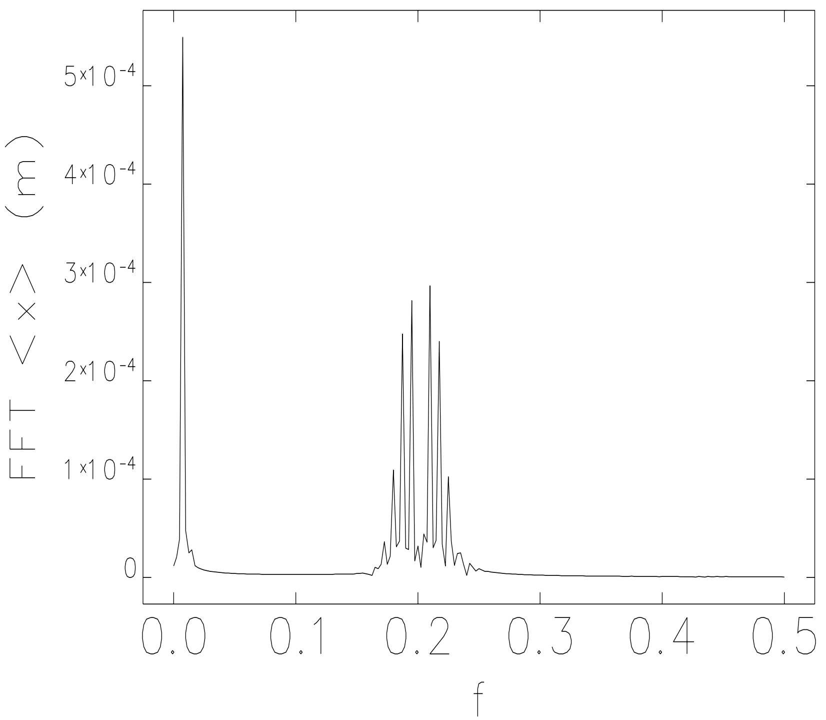
$$= \frac{\delta}{\rho}(1 - \delta + \delta^2 - \delta^3 + \cdots)$$

$$x_\delta = \eta_x(s)\delta$$

$$+ I_D(s,\delta)\sin(\nu_x(\delta)\theta + \chi(\delta)(s))(\delta - \delta^2 + \delta^3 - \cdots)$$

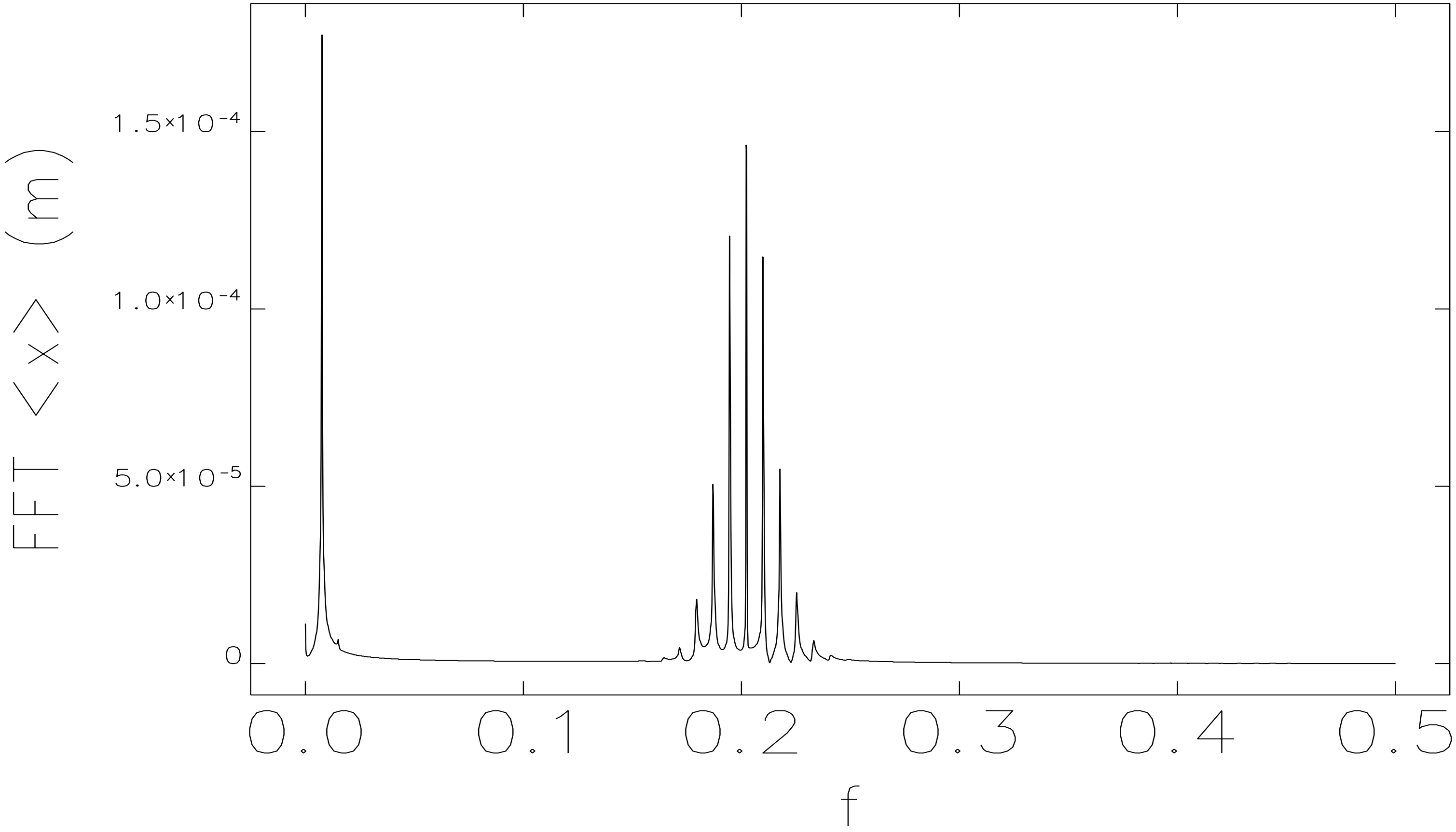
Particle simulation with ELEGANT(I)



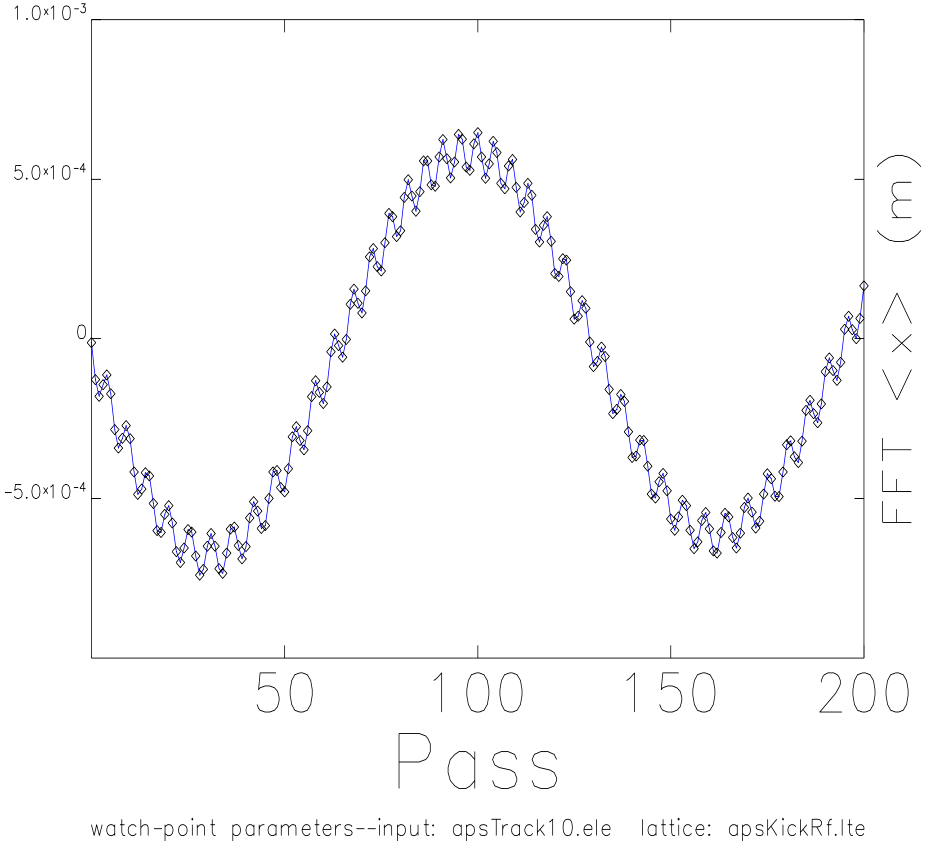


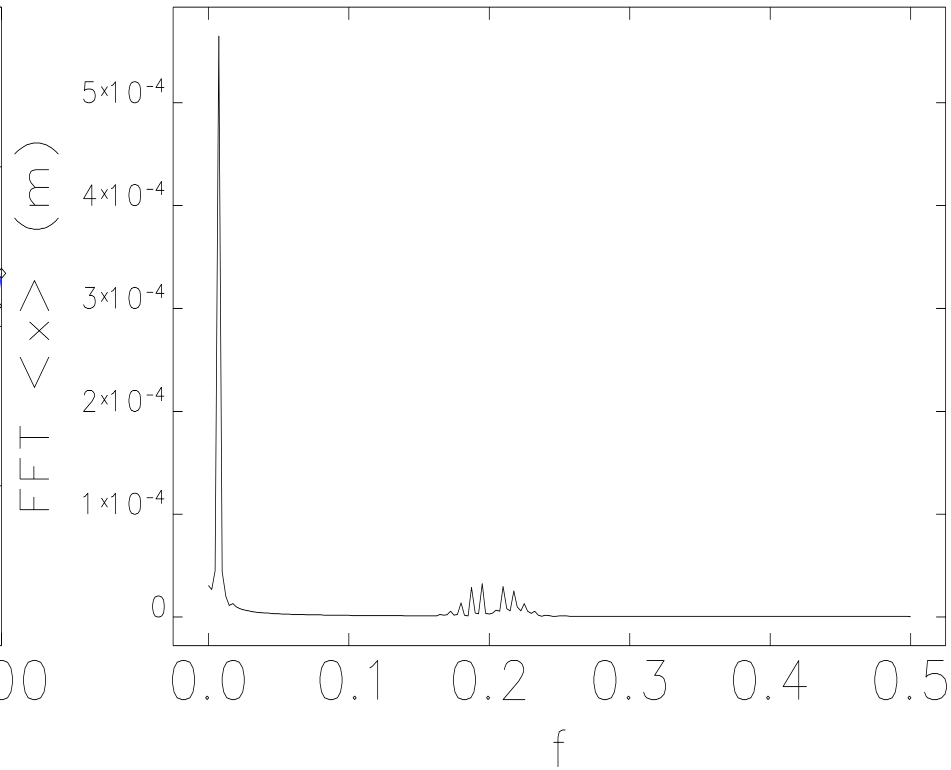
Particle on C.O. with 0.4% momentum offset

The spectrum of 4000 turns



Particle simulation with ELEGANT(II)





Particle on C.O. with 10^{o} phase kick

Experiment setup

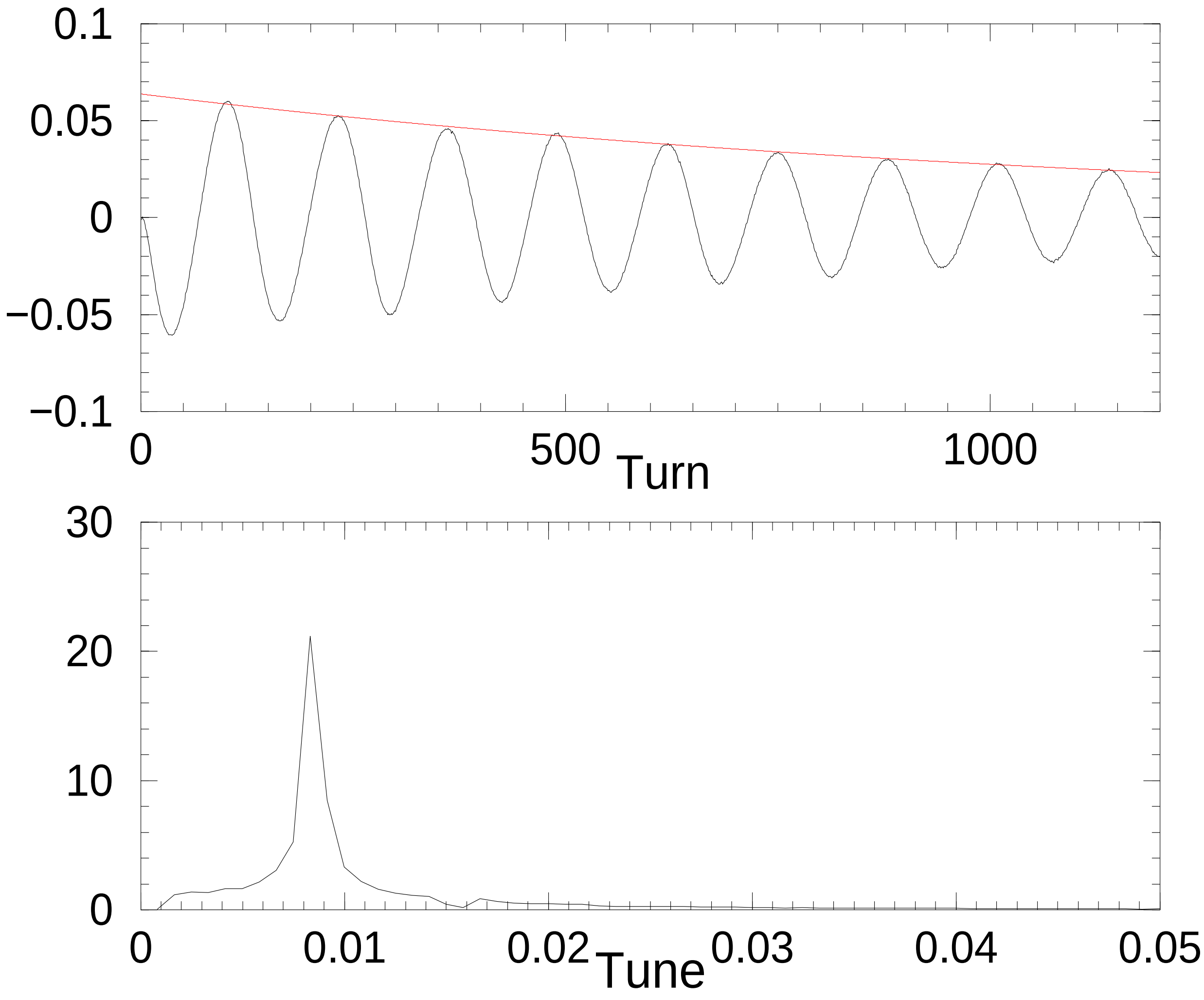
APS storage ring operation parameters

Transverse tune	36.206,19.258	Longitudinal tune	0.0083
Operation chromaticity	6.3,5.8	Revolution freq.	271.5 kHz (3.68us)
damping time	9.5(h),4.79(E) ms	RF freq.	352 MHz
Nom. Sgl bnch current	5 mA	Equil. bunch length	19.5ps(5.8mm)
Natural emittance	2.5 nm rad	Equil. mom. spread	0.096%

We give the beam a phase shift of 0~14°, with width of 200 ms. The equilibrium bunch length is about 2.5° . The horizontal displacement is about 0~1.2mm.

There are about 280 BPMs can be used for history data taking. We analyzed the data with independent component analysis.

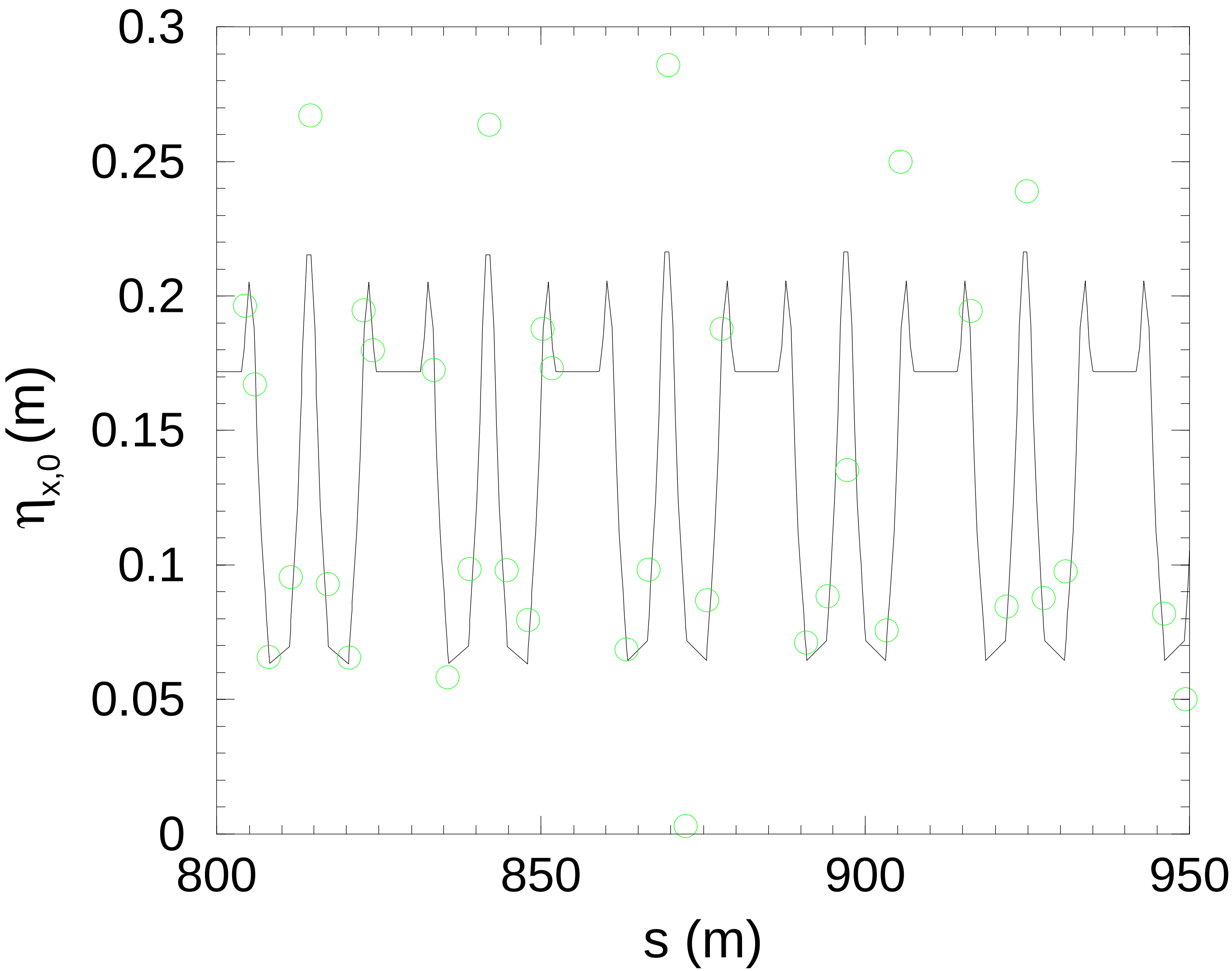
Temporal pattern of dispersion



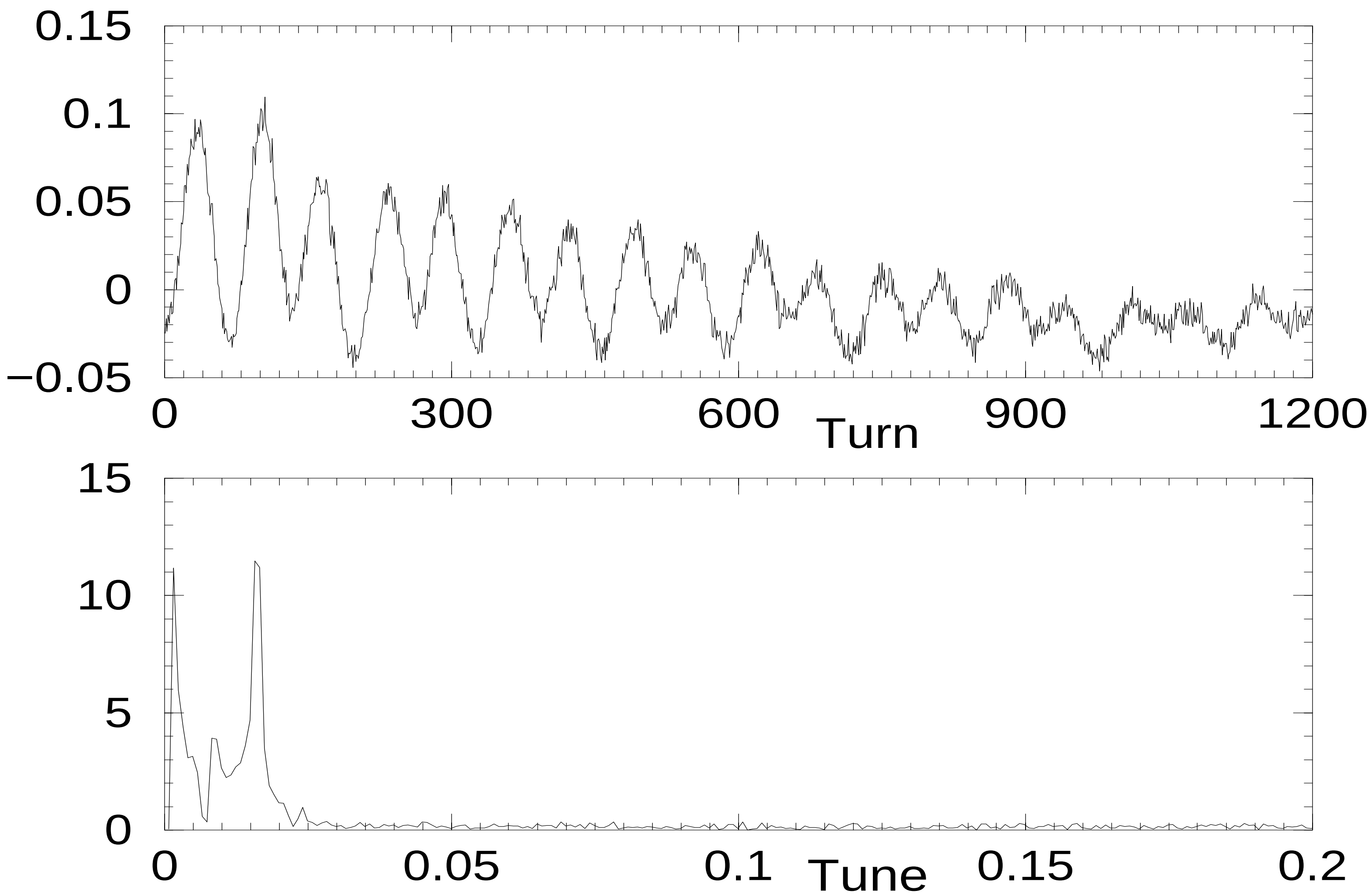
Damping time
1188 turns
1300 turns

Synchrotron tune
0.0083

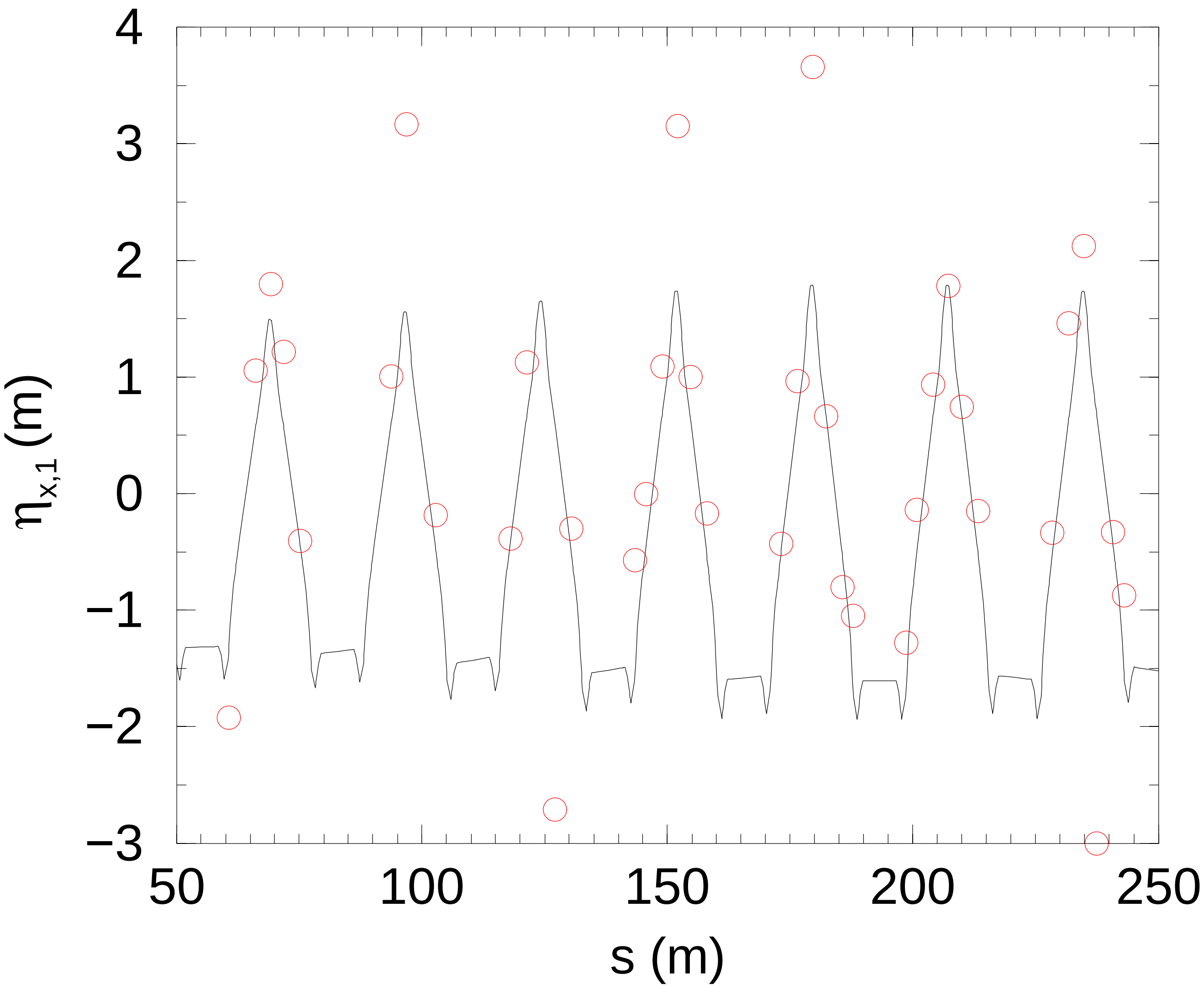
The comparison of the dispersion function



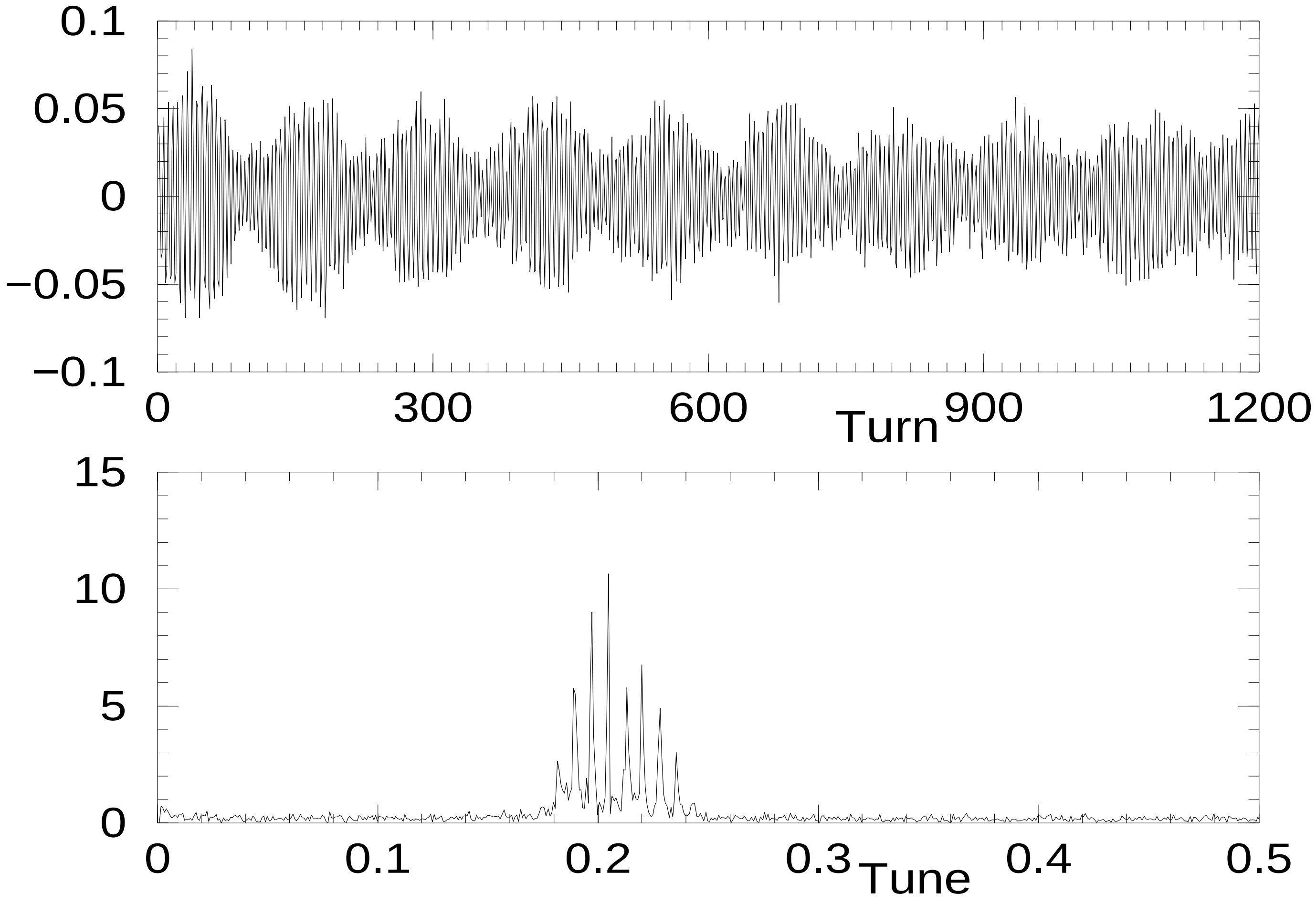
The temporal pattern of second order dispersion



The second order dispersion function



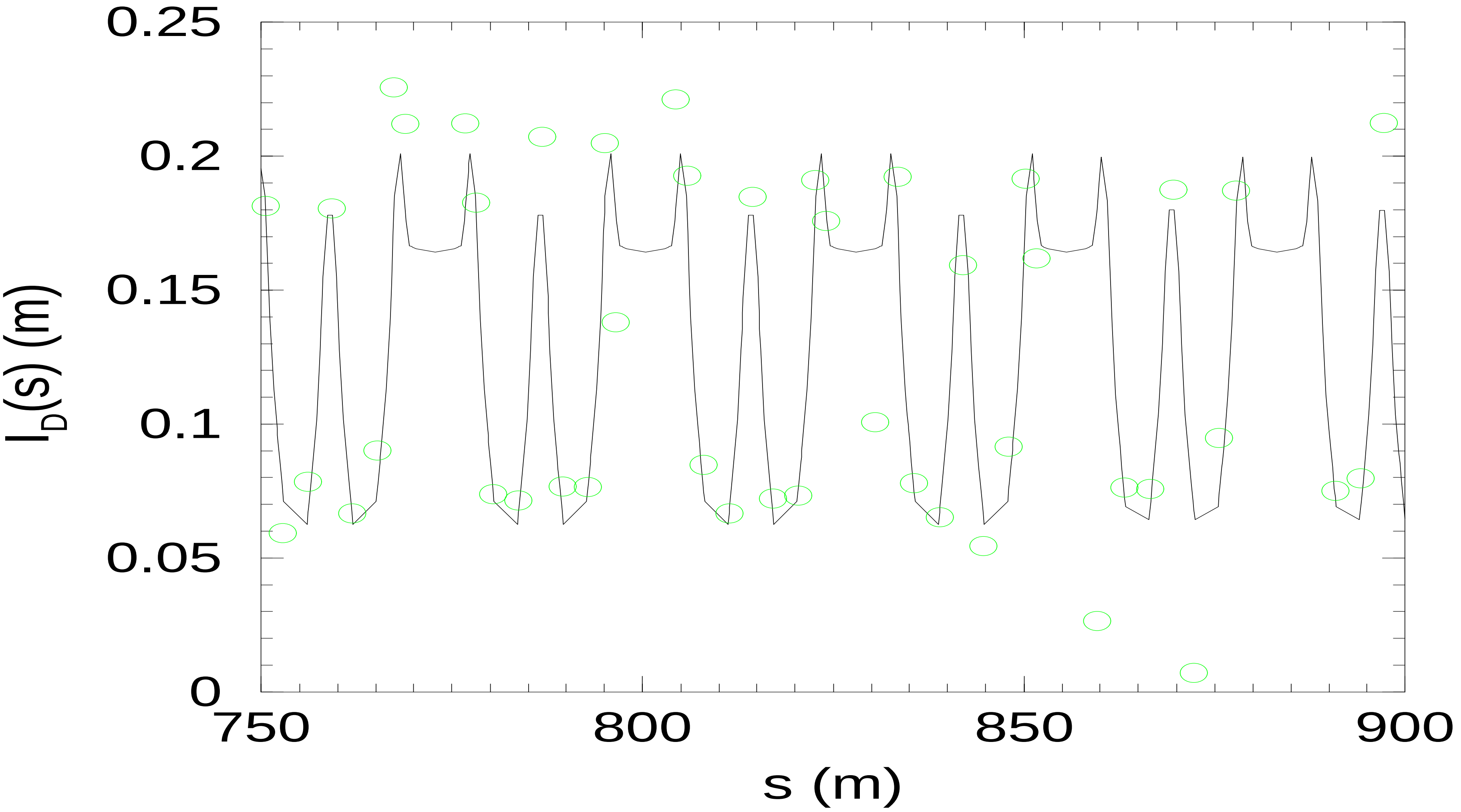
The coupling Mode



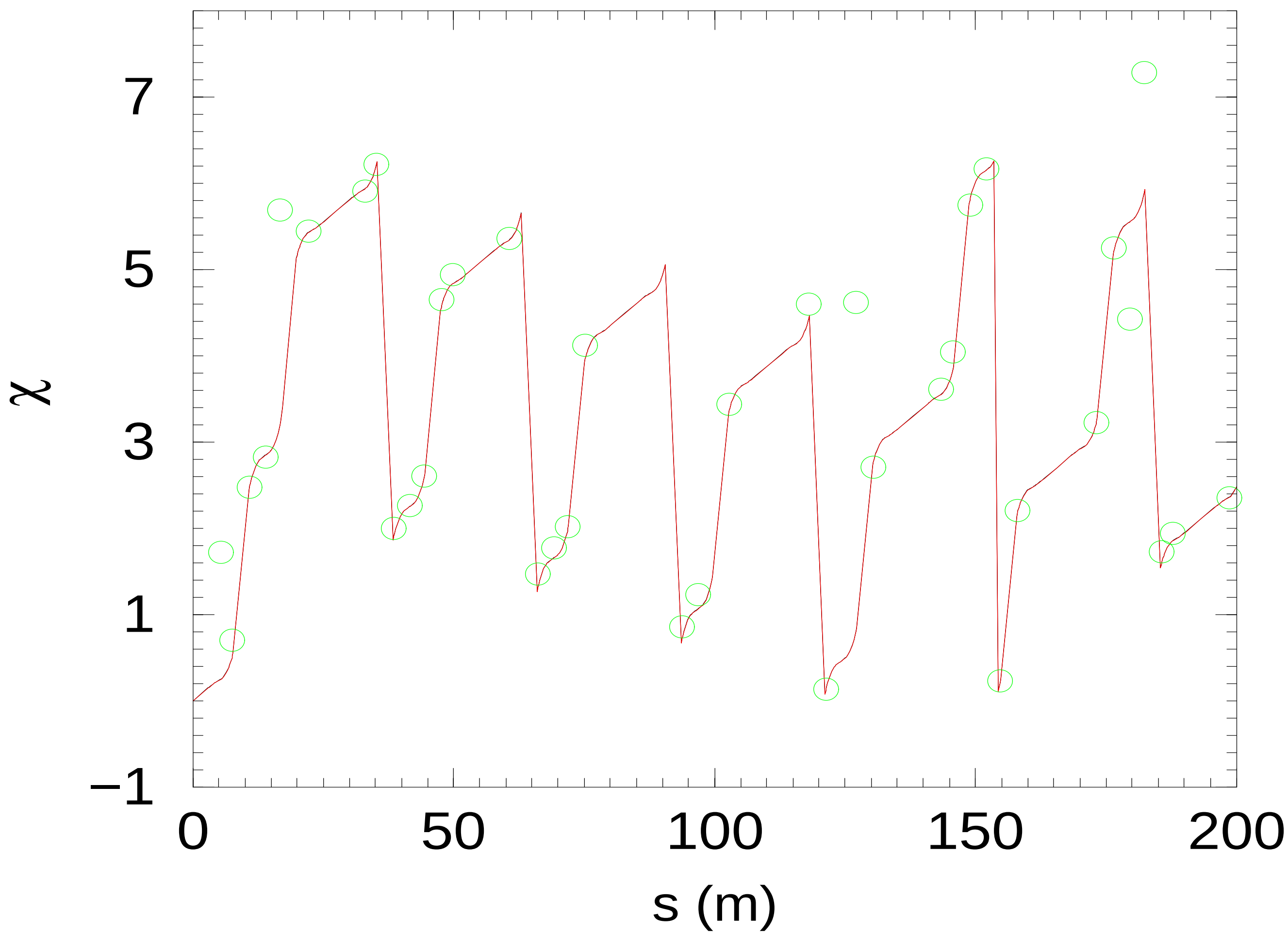
0.205

0.1975

The comparison of the l_D function



Phase advance comparison



Conclusion

- We found the analytical expression of the X-Z coupling term.
- The mode is damped in most cases, however we found it in the experiment under the critical conditions.
- Both the amplitude and phase advance agree with prediction.
- In the analysis of the experiment data, we also found the first and second order dispersion function and they all agree with model value.

The effect of the Chromaticity

$$x_\beta = x_0 \cos((\nu_x + C_x\delta)\theta + \phi_0)$$

$$\delta = \delta_0 \cos(\nu_s\theta + \psi_0)$$

$$x_\beta = x_0 \cos(\nu_x\theta + \phi_0)\begin{bmatrix} J_0(C_x\delta_0\theta) + \\ 2\sum_{k=1}^{\infty}(-1)^k J_{2k}(C_x\delta_0\theta)\cos(2k(\nu_s\theta + \psi_0)) \end{bmatrix}$$

$$+ x_0 \sin(\nu_x\theta + \phi_0)\left[2\sum_{k=0}^{\infty}(-1)^k J_{2k+1}(C_x\delta_0\theta)\cos((2k+1)(\nu_s\theta + \psi_0))\right]$$